3.1 Completed Notes

3.1: Addition and Subtraction of Whole Numbers

Definition: The whole numbers, denoted W, are the natural numbers and 0. In other words, $W = \{0, 1, 2, 3, ...\}$.

Definition: (Addition of Whole Numbers) Let A and B be two disjoint (no common elements) finite sets. If $n(A) = a$ and $n(B) = b$, then $a + b = n(A \cup B)$.

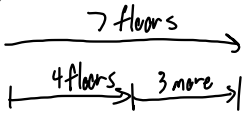
Definition: The numbers a and b are called the addends and $a + b$ is called the sum.

Addition Model 1: Representing the sum of two numbers by combining two sets is known as the set model of addition

Example: You have 3 cats and 4 dogs. How many animals do you have total?

Addition Model 2: The number line model is another way to represent addition, in which we draw a number line and represent numbers by arrows pointing right whose length is the same as the number. When we add numbers, we place the second number's line at the tip of the first number's line, and then we look at the whole length.

Example: If you were on the fourth floor of a building and moved up 3 floors, what floor are you now on?



Example: Determine which models you would use for each situation.

Your cat weighed 8 pounds and gained 2 pounds last month. How much does he weigh now?

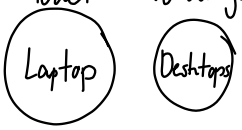
Number Line - added onto a set

You go strawberry picking and you have 6 strawberries. If you pick 5 more strawberries, how many do you have?

Number Line

You have 1 laptop and 2 desktop computers in your house. How many computers do you have total?

Set Model - Two disjoint sets



Definition: For any whole numbers a and b:

1. We say that a is less than b, denoted $a < b$, if and only if there exists a natural number k such that $a + k = b$.
2. We say that a is less than or equal to b, denoted $a \leq b$, if and only if $a < b$ or $a = b$.
3. We say that a is greater than b, denoted $a > b$, if and only if $b < a$.
4. We say that a is greater than or equal to b, denoted $a \geq b$, if and only if $b \leq a$.

Examples:

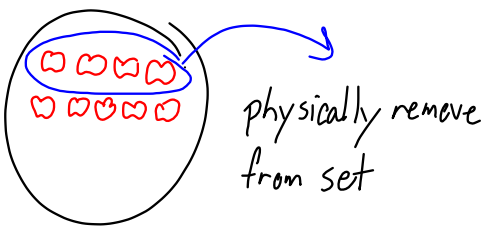
$0 < 1$ $\quad 5 \leq 5$

$\pi > 3$

Definition: For any whole numbers a and b such that $a \geq b$, $a - b$ is the unique whole number c such that $b + c = a$.

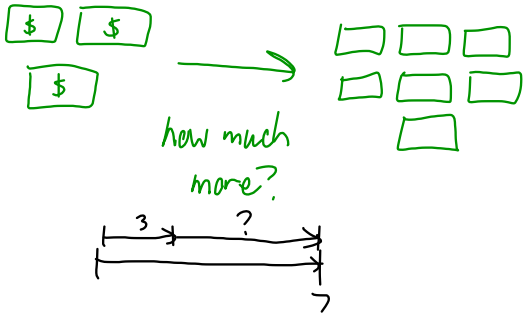
Subtraction Model 1: The Take-Away Model is used when you start with a set and remove elements. (Note: This idea could be used to define subtraction in terms of subsets and set difference.)

Example: You have 9 apples and you give 4 away to your teachers. How many apples do you have left?



Subtraction Model 2: The Missing Addend Model is used when you are determining how much is left to get to a certain number.

Example: You have 3 dollars and you want to buy a book that costs 7 dollars. How much more money do you need to buy the book?



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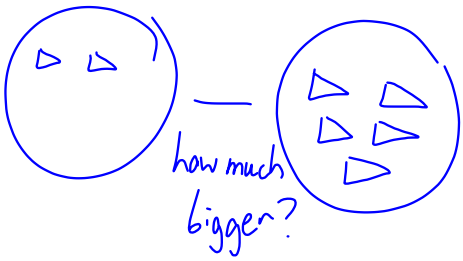
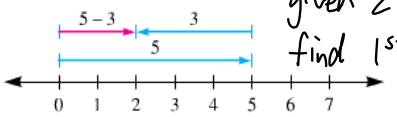
Subtraction Model 3: The Comparison Model is used when you have two sets and you want to find how many more elements are in one set than another.

Example: You ate 2 pieces of cake and your friend ate 5 pieces of cake. How many more pieces of cake did your friend eat than you did?

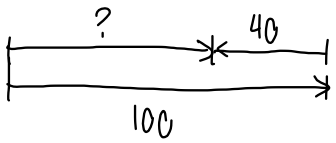


Subtraction Model 4: The Number-Line Model is used when you are given a sum and the number that was added to get there, and you want to find out what the original number was.

given 2nd
find 1st



Example: You drove 100 miles in a two hour period. If you drove 40 miles in the second hour, how many miles did you drive during the first?



Example: Determine which models you would use for each situation.

Michael is driving the 300 mile trip to Virginia. If he has driven 90 miles, how many more miles does he have to drive?

Missing Addends

Susan ate 56 donuts in two months. If she ate 6 donuts in the second month, how many donuts did you eat in the first month?

Number Line

You earned 100 points on the last test and the student next to you earned 90. How many more points did you earn than the student next to you?

Comparison

Your cat weighed 15 pounds and lost 2 pounds last month due to a much needed diet. Howmuch does he weigh now?

Take-Away

Theorem: The following properties hold for addition of whole numbers:

1. (Closure) If a and b are whole numbers, then $a + b$ is a whole number.
2. (Commutative) If a and b are whole numbers, then $a + b = b + a$.
3. (Associative) If a, b, and c are whole numbers, then $(a + b) + c = a + (b + c)$.
4. (Identity) There is a unique whole number (0 in this case), the additive identity, such that for any whole number a, $a + 0 = 0 + a = a$.

Example: Given the following sets, determine which of these properties it has.

(a) {2, 4, 6, 8, 10, ...}

Closure - Yes, any two numbers add to one in the set
Commutative - Yes, can add in either order.
Associative - Yes, can add any 3 by picking any 2
Identity - No, 0 is not in the set.

(b) {0, 2, 4, 6, 8}

Closure - No, $4+6=10$, which is not in the set
Commutative/Associative ✓
Identity - Yes, $0 \in$ set

Example: Given the following sets, determine which of these properties it has.

(c) {1, 3, 5, 7, 9, ...}

Closure - No, $1+3=4 \notin$ set.
Comm/Asso ✓
Identity - No

(d) $\{x^2 \mid x \in W\}$

$= \{0, 1, 4, 9, 16, \ldots\}$
Closure - No, $4+9=13$
Comm/Asso ✓
Identity ✓ $0 \in$ set

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Question: Which of the properties of addition of whole numbers are satisfied with subtraction of whole numbers?

Closure:

$2-3=-1 \notin W$ No

Commutative:

$2-3 \neq 3-2$ No

Associative:

$(3-2)-1=0 \quad 3-(2-1)=2$ No

Identity:

$3-0=3$ ✓

$0-3 \neq 3$ No